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The Nonlinear Stimulated Echo in the Presence of Inequivalent Spins

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A sequence of three RF pulses in the presence of magnetic field gradients at high magnetic fields ($\gtrsim$7 T) produces multiple nonlinear stimulated echoes (NOSE) at times $n\tau_1$ after the third pulse, where n is an integer and τ_1 the interval between the first two pulses. From the classical point of view, these phenomena are due to the demagnetizing field produced by the spatial modulation of the nuclear magnetization arising in the sample after the first two pulses. In this paper the behavior of NOSE for molecules containing inequivalent uncoupled spins subject to motional averaging of short-range interactions is reported. As a consequence of the demagnetizing field the interference between the magnetizations of the different spin ensembles leads to beats in the amplitude of NOSE. A semiclassical formalism describing this dependence is established and verified in test experiments with the protons in a methyl acetate sample.

Key Words: **stimulated echo; multiple echoes; NOSE; demagnetizing field.**

INTRODUCTION

Apart from the ordinary stimulated echo (STE) (*1*), the RF pulse sequence $(\pi/2)_X-\tau_1-(\pi/2)_Y-\tau_2-(\pi/4)_Y$ also produces multiple nonlinear stimulated echoes (NOSEs) (*2, 3*) in liquids, provided that the magnetic field is high and inhomogeneous enough. After the ordinary stimulated echo, the maximum of which occurs at time τ_1 after the third RF pulse, a train of NOSEs with decreasing amplitudes follows at equidistant intervals τ_1. These echoes are a consequence of the coherence evolution while the magnetization is spatially modulated along the magnetic field gradient direction.

In previous experiments the so-called ''multiple spin echoes'' were already observed after two 90° RF pulses (*4–8*). Their origin could be explained based on the classical demagnetizing field (*7, 9*). This field is a consequence of direct magnetic dipole–dipole interaction between distant spins which is not averaged out by translational or rotational diffusion.

The classical treatment is opposed by the macroscopic quantum-theoretical approach trying to describe *short-* as well as *long-range* dipolar interactions on a purely quantum-mechanical basis (*10–12*). The spin system considered in this case extends from the molecular scale up to the dimension of the whole sample. It is clear that the macroscopic quantum theory must be correct in principle. However, as in other fields of physics, it is more useful to consider the classical limit for effects on length scales far beyond those of atoms and molecules (*13, 14*).

This fact has prompted us to employ a semiclassical formalism (*3*) for the description of multiple two-pulse as well as three-pulse echoes (*1, 3*). That is, spin interactions taking place on atomistic length scales are treated using the Liouville/von Neumann equation, whereas all long-range effects are classically accounted for by considering the demagnetizing field arising from the spatially modulated magnetization.

Note that the length scale limit is well defined: Depending on the magnetic field gradient and the RF pulse spacing in a two- or three-pulse experiment, the magnetization is modulated under practical circumstances with ''wavelengths'' on the order of micrometers or longer. Far below this limit, all nonlinear effects can be neglected. The evolution of spin coherence occurs as predicted by the Liouville/von Neumann equation for frequency offsets depending linearly on the magnetic field gradients. On much longer length scales the spatial modulation of the classical demagnetizing field leads to nonlinear relations between the frequency offsets and the field gradients, and hence to multiple echo phenomena.

The introduction of long-range classical demagnetizing fields into the standard spin operator formalism for the treatment of spin-coherence evolution avoids the need to solve complicated coupled differential equations in the form of modified Bloch equations. On the other hand, the gigantic spin systems inherent to the macroscopic quantum theory approach need not be considered. In this way, NMR observables can easily be evaluated without loss of accuracy.

In our previous papers dealing with NOSE phenomena (*2, 3*), only one spin species (e.g., protons) with a certain

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chemical shift was considered. Let us now assume a second ensemble of spins with a different resonance frequency. In particular this may be due to a different chemical shift. The two inequivalent spin species will be designated by the numbers 1 and 2. Indirect spin–spin couplings are neglected. The partial magnetizations and chemical shifts are M_1, M_2 and $\delta_1 B_0$, $\delta_2 B_0$, respectively. The demagnetizing fields acting on the two spin species can be written as (*4, 15*)

$$\mathbf{B}_{d1} = \mu_0\left(M_{z1} + \frac{2}{3}M_{z2}\right)\hat{z},$$

$$\mathbf{B}_{d2} = \mu_0\left(M_{z2} + \frac{2}{3}M_{z1}\right)\hat{z}. \qquad [1]$$

The unit vector $\hat{z}$ is given by the gradient field direction, which is assumed to be parallel to the external main magnetic field $\mathbf{B}_0$. Note that expression [1] is valid only in the case of chemical shift frequency offsets of spins 1 and 2 differing by substantially more than γB_d, that is, the transverse demagnetizing field component from spin species 1 has no effect on the coherence evolution of spins 2 and vice versa (*4, 15*).

Furthermore, we assume that the field gradient G is so strong that the demagnetizing field is much less than the magnetic field variation in the sample of length l. That is, $\mu_0 M_0/Gl \ll 1$ (*7*), where M_0 is the equilibrium magnetization in the sample. Using the semiclassical approach of our previous paper (*3*), we will show in the following that the amplitude of the nonlinear stimulated echo in two-component samples oscillates with the difference frequency corresponding to the two chemical shifts. As a matter of course, many-component samples with more than two spin species can be treated using the same principles.

THEORY

In the following we consider two inequivalent chemical groups with uncoupled homonuclear spins ($J = 0$). The equilibrium magnetization is then composed of the contributions of either spin ensemble,

$$M_0 = M_{01} + M_{02}, \qquad [2]$$

where $M_{01} = kM_0$ and $M_{02} = (1 - k)M_0$, and k represents the fraction of spins 1. In the frame of the present treatment we do not consider irreversible effects such as relaxation and translational diffusion. As concerns relaxation in the pulse intervals, this means $\tau_1 \ll T_2$ and $\tau_2 \ll T_1$. We also assume that the magnetic field is inhomogeneous enough to spoil all coherences in the τ_2 interval, that is, $\tau_2 \gg T_2^*$.

The RF pulses are assumed to be ''hard'' so that they excite the sample homogeneously irrespective of the local resonance frequency offsets. Just before the first RF pulse at time ($t = 0^-$) we are dealing with thermal equilibrium, and the reduced density operator reads in the high-temperature approximation

$$\sigma(0^-) = I_{1z} + I_{2z}, \qquad [3]$$

where I_{1z}, I_{2z} represents the longitudinal components of the spin operators corresponding to spins 1 and 2, respectively. The evolution between the first and the second RF pulse takes place in the external magnetic field B_0 subject to the constant gradient G assumed along the z direction. The local frequency offsets are then

$$\Omega_1 = \gamma[\delta_1 B_0 + Gz],$$
$$\Omega_2 = \gamma[\delta_2 B_0 + Gz], \qquad [4]$$

where $\delta_1 B_0$ and $\delta_2 B_0$ represent the chemical shifts of the two components, and Gz is the gradient-induced offset at the position z. In the frame rotating with the angular frequency $\omega_0 = \gamma B_0$ the reduced density operator just before the second pulse ($t = \tau_1^-$) becomes

$$\sigma(\tau_1^-) = I_{1y}\cos\,\Omega_1\tau_1 + I_{1x}\sin\,\Omega_1\tau_1 + I_{2y}\cos\,\Omega_2\tau_1 + I_{2x}\sin\,\Omega_2\tau_1. \qquad [5]$$

Following the strategy of this work we distinguish between short- and long-range spin couplings to be treated quantum mechanically and classically, respectively. Any influence of the former is neglected. Classical long-range phenomena, that is, the consequence of the demagnetizing field, do not yet matter in this interval, because ideally there is no longitudinal magnetization, and transverse magnetization effects (''radiation damping'') are neglected.

The second RF pulse, $(\pi/2)_y$, transforms the reduced density operator into

$$\sigma(\tau_1^+) = I_{1y}\cos\,\Omega_1\tau_1 + I_{1z}\sin\,\Omega_1\tau_1 + I_{2y}\cos\,\Omega_2\tau_1 + I_{2z}\sin\,\Omega_2\tau_1. \qquad [6]$$

Note that this density operator corresponds to spatially modulated transverse as well as longitudinal magnetizations. The spatially modulated demagnetizing field following from this magnetization pattern has no effect on the density operator in the second pulse interval τ_2 because we have assumed that all spin coherences are spoiled at the time $t = \tau_1 + \tau_2^-$ owing to ($\tau_2 \gg T_2^*$). Taking this into account, the reduced density operator effective just before the third pulse can be written

$$\sigma(\tau_1 + \tau_2^-) = I_{1z}\sin\,\Omega_1\tau_1 + I_{2z}\sin\,\Omega_2\tau_1. \quad [7]$$

The third RF pulse, $(\pi/4)_y$, converts this into

$$\sigma(\tau_1 + \tau_2^+) = I_{1z}\frac{1}{\sqrt{2}}\sin\,\Omega_1\tau_1 + I_{2z}\frac{1}{\sqrt{2}}\sin\,\Omega_2\tau_1 - I_{1x}\frac{1}{\sqrt{2}}\sin\,\Omega_1\tau_1 - I_{2x}\frac{1}{\sqrt{2}}\sin\,\Omega_2\tau_1. \quad [8]$$

As can readily be seen, the magnetization is split into two components modulated along the z direction with the respective wavelengths. The transverse components evolve in the presence of the demagnetizing field created by the longitudinal components, whereas that of the transverse magnetizations may be neglected. Since the longitudinal magnetization distribution in this interval is assumed not to change, either by coherence evolution or by relaxation or diffusion, we can focus on the evolution of the transverse component under the passive influence of the spatially modulated demagnetizing field.

For the signals produced by free evolution after the third RF pulse, the following initial terms of the reduced density operator are relevant:

$$\sigma(\tau_1 + \tau_2^+) = -I_{1x}\frac{1}{\sqrt{2}}\sin\,\Omega_1\tau_1 - I_{2x}\frac{1}{\sqrt{2}}\sin\,\Omega_2\tau_1. \quad [9]$$

The single-quantum coherences evolving after the third RF pulse can be described as a rotation of the I_{ix} $(i = 1, 2)$ spin operators around the z axis by the angles

$$\varphi_1(z, t) = \gamma[\delta_1 B_0 + Gz + B_{d1}]t = \Omega_1 t + \gamma B_{d1} t,$$
$$\varphi_2(z, t) = \gamma[\delta_2 B_0 + Gz + B_{d2}]t = \Omega_2 t + \gamma B_{d2} t, \quad [10]$$

where B_{d1}, B_{d2} represent the demagnetizing fields seen by spins 1 and 2, respectively. These two fields can be easily computed using Eq. [1] as

$$B_{d1} = k\mu_0 M_0 \sin\,\Omega_1\tau_1 + \frac{2}{3}(1 - k)\mu_0 M_0 \sin\,\Omega_2\tau_1 \quad [11]$$

and

$$B_{d2} = (1 - k)\mu_0 M_0 \sin\,\Omega_2\tau_1 + \frac{2}{3}k\mu_0 M_0 \sin\,\Omega_1\tau_1. \quad [12]$$

Therefore, the reduced density operator evolves after the third radiofrequency pulse according to

$$\sigma(\tau_1 + \tau_2^+ + t) = -[I_{1x}\cos\,\varphi_1(t) - I_{1y}\sin\,\varphi_1(t)]\frac{1}{\sqrt{2}}\sin\,\Omega_1\tau_1 - [I_{2x}\cos\,\varphi_2(t) - I_{2y}\sin\,\varphi_2(t)]\frac{1}{\sqrt{2}}\sin\,\Omega_2\tau_1. \quad [13]$$

The total complex transverse magnetization $M_+ = M_x + iM_y$ can be computed as a contribution of the two components, i.e., $M_+ = M_{1+} + M_{2+}$. These partial complex magnetizations at a time τ after the third pulse are

$$\begin{aligned} M_{1+}(z, \tau) &= M_{1x} + iM_{1y} \\ &= \mathrm{Tr}\{\sigma(\tau_1 + \tau_2 + \tau)(I_{1x} + iI_{1y})\} \\ &= i\frac{\sqrt{2}}{4}kM_0[e^{-i[\varphi_1(\tau) - \Omega_1\tau_1]} - e^{-i[\varphi_1(\tau) + \Omega_1\tau_1]}] \end{aligned} \quad [14]$$

and

$$\begin{aligned} M_{2+}(z, \tau) &= M_{2x} + iM_{2y} \\ &= \mathrm{Tr}\{\sigma(\tau_1 + \tau_2 + \tau)(I_{2x} + iI_{2y})\} \\ &= i\frac{\sqrt{2}}{4}(1 - k)M_0 \\ &\quad \times [e^{-i[\varphi_2(\tau) - \Omega_2\tau_1]} - e^{-i[\varphi_2(\tau) + \Omega_2\tau_1]}], \end{aligned} \quad [15]$$

respectively.

In order to evaluate the signal of the whole sample, we must take the average of the complex magnetization over all z positions. Strong gradients lead to an equipartition of the phase angles $\varphi_i(z) \pm \Omega_i\tau_1$ $(i = 1, 2)$ so that the averages of the terms just shown vanish unless they are independent of position. At all times when the latter situation arises, multiple nonlinear stimulated echoes are expected. The condition for these refocusing times is found by expanding the exponential terms in Eqs. [14] and [15] in terms of Bessel functions (*3*, *11*):

$$e^{i\xi\sin\varphi} = \sum_{m=-\infty}^{+\infty} J_m(\xi)e^{im\varphi}. \quad [16]$$

Exploiting the properties of Bessel functions of integer order $J_m(\xi)$ (*16*),

$$J_{m-1}(\xi) - J_{m+1}(\xi) = 2\frac{d}{d\xi}[J_m(\xi)], \quad [17]$$
$$J_{-m}(\xi) = (-1)^m J_m(\xi)$$

leads to

$$e^{-i[\varphi_1(\tau)\pm\Omega_1\tau_1]} = \sum_{n,m=-\infty}^{+\infty} J_{n\mp1}(\xi_1'(\tau)) \times J_m(\xi_2'(\tau)) e^{-i\gamma B_0[(n\tau_1+\tau)\delta_1+m\tau_1\delta_2]} e^{-i[(m+n)\tau_1+\tau]\gamma Gz},$$

$$e^{-i[\varphi_2(\tau)\pm\Omega_2\tau_1]} = \sum_{n,m=-\infty}^{+\infty} J_{n\mp1}(\xi_1''(\tau)) \times J_m(\xi_2''(\tau)) e^{-i\gamma B_0[(n\tau_1+\tau)\delta_2+m\tau_1\delta_1]} e^{-i[(m+n)\tau_1+\tau]\gamma Gz}. \quad [18]$$

The arguments of the Bessel functions are

$$\xi_1'(\tau) = \frac{1}{\sqrt{2}} k\gamma\mu_0 M_0\tau,$$

$$\xi_2'(\tau) = \frac{2}{3\sqrt{2}} (1-k)\gamma\mu_0 M_0\tau,$$

$$\xi_1''(\tau) = \frac{1}{\sqrt{2}} (1-k)\gamma\mu_0 M_0\tau,$$

$$\xi_2''(\tau) = \frac{2}{3\sqrt{2}} k\gamma\mu_0 M_0\tau. \quad [19]$$

Substituting these relationships in Eqs. [14] and [15] and averaging over the z position (denoted by $\langle\cdots\rangle$) results in the value zero unless $\tau = -(n+m)\tau_1$, $(\tau > 0)$, where n, m are integers. These are the intervals after the third RF pulse after which multiple nonlinear stimulated echoes appear (*3*). Their amplitudes can be evaluated from the complex transverse magnetizations

$$\langle M_{1+}(\tau = -(n+m)\tau_1)\rangle = -i\frac{1}{\sqrt{2}} kM_0 \sum_{n,m=-\infty}^{+\infty} \left[\frac{\partial}{\partial\xi_1'} J_n(\xi_1')\right] \times J_m(\xi_2') e^{-im\tau_1(\delta_2-\delta_1)\gamma B_0} \quad [20]$$

and

$$\langle M_{2+}(\tau = -(n+m)\tau_1)\rangle = -i\frac{1}{\sqrt{2}} (1-k)M_0 \sum_{n,m=-\infty}^{+\infty} \left[\frac{\partial}{\partial\xi_1''} J_n(\xi_1'')\right] \times J_m(\xi_2'') e^{-im\tau_1(\delta_1-\delta_2)\gamma B_0}, \quad [21]$$

respectively.

The first nonlinear stimulated echo appears at time $\tau = 2\tau_1$ after the third pulse, so that $n + m = -2$ in Eqs. [20] and [21]. For intervals, τ_1, of the order of tens of milliseconds, that is, ξ_i', $\xi_i'' \ll 1$ $(i = 1, 2)$, the Bessel function expansion [16] may be restricted to the first order with reasonable accuracy. The average complex magnetization components are then found to be

$$\langle M_{1+}(2\tau_1)\rangle = -i\frac{1}{4\sqrt{2}} kM_0 \times [\xi_1'(2\tau_1) + \xi_2'(2\tau_1) e^{i(\delta_2-\delta_1)\gamma B_0\tau_1}] \quad [22]$$

and

$$\langle M_{2+}(2\tau_1)\rangle = -i\frac{1}{4\sqrt{2}} (1-k)M_0 \times [\xi_1''(2\tau_1) + \xi_2''(2\tau_1) e^{i(\delta_1-\delta_2)\gamma B_0\tau_1}]. \quad [23]$$

The NMR signal arises from both magnetization components so that the amplitude of the nonlinear stimulated echo is expected to be proportional to

$$\langle M_+(2\tau_1)\rangle = \langle M_{1+}(2\tau_1)\rangle + \langle M_{2+}(2\tau_1)\rangle = -i\frac{M_0^2}{4}\gamma\mu_0\tau_1 \{k^2 + (1-k)^2 + \frac{4}{3} k(1-k)\cos[(\delta_2-\delta_1)\gamma B_0\tau_1]\}. \quad [24]$$

The amplitudes of the higher-order nonlinear stimulated echoes can be computed analogously. However, the amplitudes will decrease rapidly with increasing order. Analyzing the preceding expression we can observe that in a two-component system, the amplitude of nonlinear stimulated echo de-

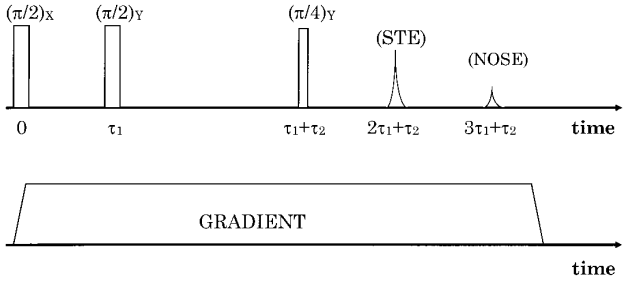

FIG. 1. Sequence of three RF pulses generating a stimulated echo (STE) and a nonlinear stimulated echo (NOSE) in an inhomogeneous magnetic field. In the pesent study, a gradient $G = 0.7$ mT/m was applied along the z direction. The widths of the $\pi/2$ and $\pi/4$ RF pulses were 18 and 9 μs, respectively. A phase cycle was used in order to avoid overlap of the classical stimulated echo and NOSE.

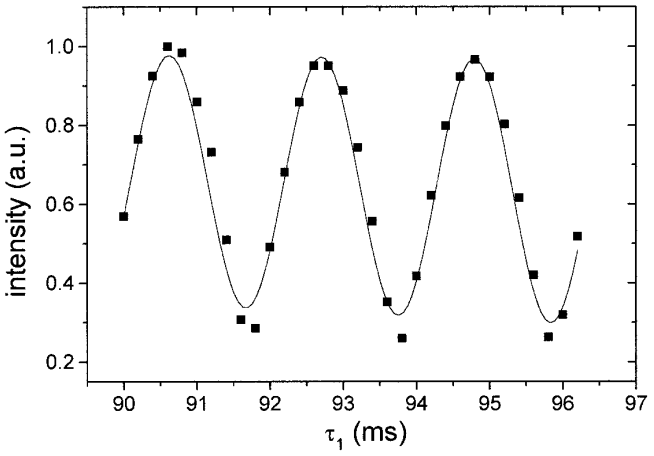

FIG. 2. NOSE amplitude versus τ_1 for a fixed interval $\tau_2 = 750$ ms in methyl acetate. The theoretical curve, Eq. [24], is also represented (continuous line).

pends in a sinusoidally modulated manner on the τ_1 interval. The frequency of these oscillations is proportional to the chemical shift difference $(\delta_2 - \delta_1)B_0$ between the two spin species. In the absence of relaxation and diffusion, these amplitude beats ideally never cease. It is important to note that this temporal modulation is a phenomenon solely based on the existence of the spatially modulated demagnetizing field, whereas short-range direct as well as indirect spin couplings are not considered here. The demagnetizing-field effects are clearly to be distinguished from J modulations which are well known with any sort of spin echoes.

EXPERIMENTS AND RESULTS

The experiments were carried out on a Bruker MSL 300 spectrometer ($B_0 \cong 7$ T). A 5-mm sample tube was filled to a height of 25 mm with methyl acetate, CH_3COOCH_3, so that $k = 1/2$. The chemical shift difference between the two chemical groups under consideration corresponds to $\Delta\nu \cong 490$ Hz at 7.07 T. The sample temperature was 293 K.

Figure 1 represents the general scheme used for generating stimulated and nonlinear stimulated echoes. The field gradient in our experiments was $G = 0.7$ mT/m along the z direction. The first evolution interval τ_1 was varied between 90 and 96.2 ms with an increment of 0.02 ms. The second interval τ_2 was chosen to be 750 ms in order to completely defocus the coherences during this interval. The RF pulse width for a flip angle of $\pi/2$ was 18 μs. A suitable pulse and receiver phase cycle was chosen in order to avoid the overlap of the conventional stimulated echo and the nonlinear stimulated echo. The particular phases chosen are not restrictive for the appearance of NOSE.

Figure 2 represents the NOSE amplitude as a function of τ_1 for a fixed value of $\tau_2 = 750$ ms. The theoretical curve, Eq. [24], is also represented in good accordance with the experimental data. *A posteriori* this justifies the neglect of diffusion and longitudinal relaxation in the foregoing treatment.

DISCUSSION AND CONCLUSIONS

The effect of a second spin species on the nonlinear stimulated echo was treated theoretically. The predicted features were verified experimentally. The treatment is based on a semiclassical theory in which long-range dipolar interactions are considered classically in the form of the demagnetizing field, whereas the coherence pathway in the course of the pulse sequence is evaluated quantum mechanically. This formalism permitted us a perfect description of all experimental NOSE characteristics found with the two proton species in methyl acetate. In particular, the NOSE amplitude as a function of τ_1 is modulated sinusoidally with a frequency corresponding to the chemical-shift difference of the two inequivalent spin species.

An analogous dependence was already observed for ordinary multiple spin echoes in a two-component system (*5*), and the indirect detection of the second component was suggested in (*6*). Such a detection scheme, which is based on the influence of one chemical component on the coherence evolution of another one via the demagnetizing field, requires neither that the two spin species be short-range coupled nor that they be located in the same molecule. This method may be reconsidered in the light of the nonlinear stimulated echo described earlier. Anyway, demagnetizing-field effects in multicomponent samples are an obvious reality and should not be ignored. Such effects combined with the influence of intramolecular J coupling have recently been considered in Ref. (*17*).

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